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COMMENT

Percolation in an edge

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Abstract. Percolation in a three-dimensional lattice, bounded by two free plane surfaces meeting at an angle α, is studied within a real space renormalisation scheme. The edge critical behaviour at various surface and bulk transitions is analysed. We do not find evidence for a first-order edge transition at higher values of α, as has recently been observed for an Ising model at an edge. Dependence of the edge scaling power y_e on the opening angle α has been confirmed.

Critical behaviour in semi-infinite systems has attracted much attention during the last few years [1]. These systems, besides being of practical experimental importance, present many novel features. Various theoretical techniques, like mean-field theory [1, 2], field-theory methods [3, 4] and real space renormalisation groups (RSRG) [5-9] have been applied to study such systems. In a semi-infinite system, e.g., the three-dimensional Ising model with a free surface, four types of phase transitions can occur: ordinary, surface, extraordinary, and special multicritical, depending on the parameters of the system.

Recently some attention has been paid to the critical behaviour in a semi-infinite Ising model bounded by two plane surfaces which meet at an edge [10-12]. Larsson [11] has treated an Ising model at an edge with an angle α between the intersecting surfaces. Using RSRG methods he found that, for angles $\alpha > \alpha^{**} \approx 297°$, for the special surface-bulk transition, the edge fixed point diverges and the edge magnetic scaling exponent becomes 1, equal to the dimensionality of the edge, thus indicating a first-order edge transition. Larsson also studied the edge critical behaviour within the Migdal-Kadanoff [13, 14] renormalisation group (MKRG) scheme for a q-state Potts model and observed that α^{**} grows with q. As is well known [15] the q-state Potts model includes the cases of Ising model ($q=2$) and bond percolation ($q=1$), and it was suggested [11] that a study of percolation in an edge could probably clarify the situation with regard to the first-order edge transition.

In this comment we report a RSRG study of bond percolation in a semi-infinite three-dimensional lattice bounded by two free plane surfaces meeting at an angle α. In contrast to the results for the Ising model [11] we do not find any evidence for a first-order edge transition at the special surface-bulk transition.

We consider a semi-infinite three-dimensional lattice bounded by two plane surfaces meeting at an angle α, and assume that the bonds in the bulk, surfaces and the edge have concentrations p, q and r respectively. Using a Migdal-Kadanoff [13, 14] type

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renormalisation group (RG) scheme, modified by Larsson [11] to incorporate the edge angle α, one gets the following RG recursion relations for the variables p, q and r;

$$p' = 1 - (1 - p^b)^{b^{(d-1)}} \tag{1}$$

$$q' = 1 - (1 - q^b)^b (1 - p^b)^{b^{(d-2)}} \tag{2}$$

$$r' = 1 - (1 - r^b)(1 - q^b)^{b-1}(1 - p^b)^{\lambda} \tag{3}$$

where

$$\lambda = (b-1)\left(\frac{(b+1)\alpha}{2\pi} - \frac{1}{2}\right) \tag{4}$$

and b is the RG scaling factor, which in the present calculation we take as 3. (We choose $b = 3$ to incorporate the geometrical asymmetry of the system at the edge; for more details see [12]. However, it is expected that b close to unity, as used by Larsson, should lead to similar results as presented here.) As discussed by Larsson [11] the value of λ, as given by equation (4), is correct for angles α which are integer multiples of $\pi/3$ or $\pi/2$. However, it can be applied to the general values of α as a reasonable approximation.

We have used the recursion relations, equations (1)-(3), to analyse the critical behaviour of the edge at various surface-bulk critical points. First, we discuss the particular case when $\alpha = \pi/2$, which shows the general features of the edge critical behaviour. The fixed points and the RG flow for the simple cubic lattice ($\alpha = \pi/2$) are shown in figure 1. The fixed points S, B1, B2, and BS represent the surface, ordinary, extraordinary, and the special transitions, respectively, for the surface-bulk system. The fixed point SE represents the case when the surface and the edge undergo a simultaneous phase transition. B3 and BSE are the edge fixed points corresponding to ordinary and special surface-bulk transitions. In terms of the percolation clusters [16], at these fixed points, even if the edge itself does not have an infinite cluster (which is possible only when $r = 1$), it belongs (at least a fraction of its bonds) to an infinite cluster extending either in the bulk or in the surface. Similarly at the fixed point SE the edge bonds belong to an infinite cluster in the surface, even though there

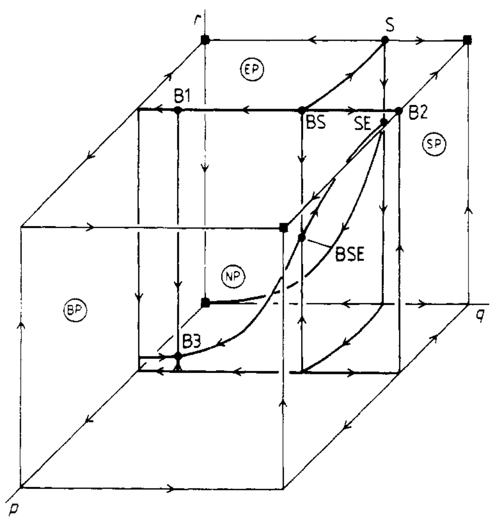

Figure 1. Fixed points and RG flow for a semi-infinite cubic lattice with an edge ($\alpha = 90°$).

are no bonds present in the bulk. (In this case the system is equivalent to two free plane intersecting surfaces inclined at an angle α.) At the three fixed points SE, B3, and BSE the edge variable r turns out to be irrelevent and the edge critical behaviour is governed either by the surface or by the bulk critical exponents, depending on the type of surface-bulk transition.

Now we discuss the edge critical behaviour for general values $0° < \alpha < 360°$. We used the edge recursion relation equation (3) to calculate the edge fixed point r^* and the edge scaling exponent y_e (which is related to the correlation length exponent ν_e through $y_e = b^{1/\nu_e}$) for this range of the values of α at the ordinary and special surface-bulk transitions. The results for the edge fixed point are shown in figure 2 and the edge scaling exponent is shown in figure 3. At the ordinary transition the edge fixed point becomes unphysical (negative) for angles $\alpha < \alpha^* \approx 38°$. This result is different from that of Larsson [11] for an Ising model where α^* is found to be 53°. For the range $38° < \alpha < 360°$ the edge fixed point and scaling exponent, at the ordinary transition, are continuously varying functions of α.

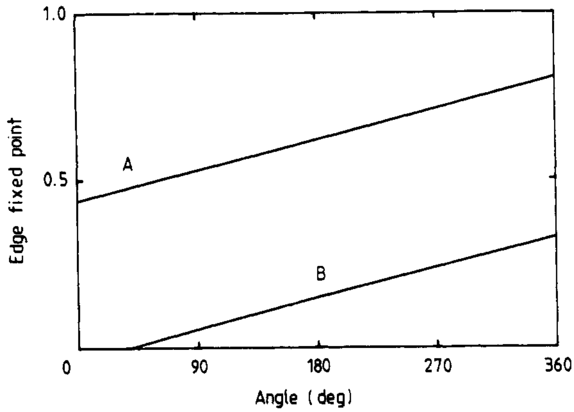

Figure 2. Variation of edge fixed point at special (A) and ordinary (B) surface-bulk transitions.

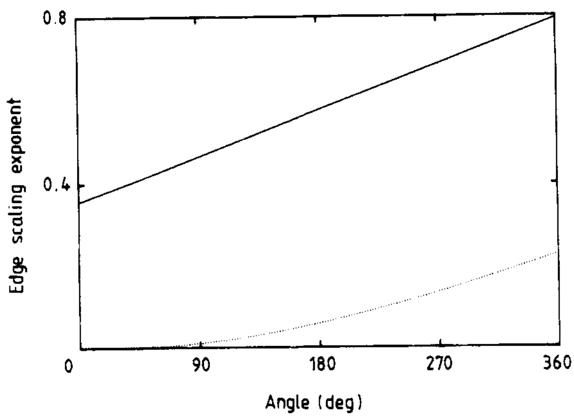

Figure 3. Variation of edge scaling exponent y_e at special (full line) and ordinary (dotted curve) transitions.

At the special transition the edge fixed point shows a very smooth behaviour and remains finite for all the values of α. This is contrary to the case of the Ising model at an edge [11] where at the special transition the edge fixed point was found to diverge for $\alpha > \alpha^{**} \approx 297°$, and the edge magnetic exponent became 1, equal to the dimensionality of the edge, indicating a first-order edge transition. In our case, for percolation in an edge, the edge exponent always remains less than one. Thus our results do not show any evidence of a first-order edge transition.

Our results for the surface critical behaviour are in agreement with the theory of Bray and Moore [17] and recent results on surface effects on percolation [18], predicting a surface-bulk cooperation for a three-dimensional semi-infinite system. On the other hand the edge does not show any cooperative behaviour and no edge phase transition curve is observed below the one-dimensional percolation threshold ($r_e = 1$), as is expected. Recently Grassberger [19] has studied the spreading of three-dimensional percolation where the seed consists of an edge in a semi-infinite system. However, it is not very clear how our exponents are related to Grassberger's surface and edge exponents z_1 and z_2, as the two situations are little different.

In conclusion we have studied percolation in an edge for various opening angles α using a RSRG method. We find that the edge always undergoes a second-order phase transition, along with the surface-bulk transition. The edge critical behaviour is governed either by the bulk or by the surface exponents. Our calculations have the usual limitations which are common with the MK type RG and perhaps a better RG scheme or a Monte Carlo simulation on such systems could throw more light.

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